Are priors responsible for cosmology favoring additional neutrino species?

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(Dated: June 28, 2011)

It has been suggested that both recent cosmological data and the results of flavor oscillation experiments (MiniBooNE and LSND) lend support to the existence of low-mass sterile neutrinos. The cosmological data appear to weakly favor additional forms of radiation in the Universe, beyond photons and three standard neutrino families. We reconsider the cosmological evidence by making the resulting confidence intervals on the additional effective neutrino species as prior-independent as possible. We find that, once the prior-dependence is removed, the latest cosmological data show no evidence for deviations from the standard number of neutrino species.

I. INTRODUCTION

The standard model for particle physics has three massless neutrinos. Beyond the standard model physics is needed to give neutrinos a non-zero mass and hence explain measurements of neutrino oscillations. It is therefore reasonable, when considering extensions of the standard model, to explore the possibility of more than three neutrinos species. A deviation from the standard number of neutrino species $N_\nu = 3$ affects the expansion history of the early universe, which in turn modifies nucleosynthesis constraints. Observations of the Cosmic Microwave Background (CMB) and of large-scale structure also probe the radiation density at matter-radiation equality and at recombination; this physical energy density in relativistic particles can be expressed in terms of the energy density in photons (which is highly constrained by the measurement of the CMB temperature) and the effective number of neutrino species N_{eff} which, even in the standard scenario [28], differs from the number of neutrino species N_ν to account for QED effects and for neutrinos being not completely decoupled during electron-positron annihilation (see [1] for a review and refs. therein). Any light particle that does not couple to electrons, ions and photons, such as a sterile neutrino, will act as an additional relativistic species. Deviations from the standard N_{eff} value can also arise from decay of dark matter particles, quintessence, light axions or other exotic models [2]. Nucleosynthesis and CMB constraints on N_ν rely on completely different physics and correspond to very different epochs in the Universe's evolution, providing an overall consistency check.

Recently, there has been renewed interest in the deviations for the standard number of neutrino species, see e.g., [3]. Extra neutrinos could explain for example the LSND results and some of the latest MiniBoone results [4–6]. The anomalous X-ray narrow emission feature from the dwarf spheroidal galaxy Willman 1 has also been explained in terms of sterile neutrinos [7].

The current situation is summarized in Ref. [8]. The authors consider the latest cosmological constraints on the number of neutrino species and their possible connection with particle physics experiments. They find that recent cosmological data, alone or in combination with big-bang nucleosynthesis constraints, weakly favor extra radiation in the Universe beyond photons and ordinary neutrinos, lending support to the existence of low-mass sterile neutrinos. They find that the amount of additional radiation corresponds to one extra N_{eff}. An extra species of low mass sterile neutrinos would fit both cosmological data and flavor oscillation results.

Note that in some analyses of cosmological data the standard N_{eff} value is ruled out at better than 95% confidence, e.g. [9], while in other analyses it is not, e.g. [10]. More recently, the latest CMB temperature data, including high-resolution measurements of the damping tail, were combined in Ref. [11] to show that this preference for extra neutrino species in the cosmological data comes from their contribution to the expansion rate prior to recombination. This analysis also rules out the standard neutrino number, N_{eff}, at the 95% and 98.4% confidence level with data sets consisting of CMB data and of a combination of CMB, Large-scale structure and the Hubble constant respectively. The physical interpretation offered is that a larger value of N_{eff} produces an increased Silk damping at high ℓ and this seems to improve the fit to the data.

Given the importance of the consequences of a deviation from the standard effective number of neutrino species and the reported hints for extra radiation from precision cosmological data, we reconsider the cosmological constraints on N_{eff}.

II. METHOD

Early work ([9, 10, 13–16] and references therein) as well as more recent papers [11, 12] have reported constraints derived in the Bayesian framework. In this framework constraints are obtained from the posterior distribution, which is marginalized over the uninteresting parameters to derive the desired confidence intervals. While this approach is extremely powerful and enables one to do statistical inference, the posterior distribution depends on the priors chosen for each of the parameters, and this choice is often arbitrary. Furthermore, when deriving confidence intervals, the marginalization procedure weights parameters by the posterior volume, rather than just by how well the parameters fit the data. When

considering a high-dimensional parameter space, especially if the data-sets are not highly constraining, the reported confidence regions on a parameter can depend strongly on the prior choice and on the resulting prior volume effects.

Moreover the widely used software "Get Dist" – part of the CosmoMC package [17]– provides by default the Bayesian central credible interval while the minimal credible interval may be more suitable for statistical inference. As noted in [18] these two confidence intervals differ significantly if the posterior is skewed and the effect is large especially for $N_{\rm eff}$ if the data-sets chosen are not very constraining. As more data sets are considered, degeneracies are reduced, the posterior become nearly Gaussian and the two confidence intervals become closer.

If a result of cosmological analysis, such as any evidence for $N_{\rm eff} > 3.04$, turns out to be driven by any of the effects above, it should be interpreted with extra care.

The dependence on the (arbitrary) choice for the prior, on the posterior volume effect in marginalization and the ambiguity of central vs minimal credible interval can be removed by making the analysis as prior-independent as possible.

In the frequentist approach the generalized likelihood ratio is widely used to report confidence intervals. The likelihood ratio is a well established technique to compute frequentist confidence interval when only one parameter is considered, but it lacks of a feature equivalent to Bayesian marginalization when dealing with high-dimensional (multi-parameters) problems. The generalized (or profile) likelihood ratio on the other hand includes such a feature.

Fortunately, it is possible to extract profile likelihood ratio-based confidence intervals from the standard output of cosmological analyses implemented with Markov Chain Monte Carlo methods [10].

To perform our Markov Chain Monte Carlo analysis, we use the publicly available CosmoMC package [17]. The output from a CosmoMC analysis for a given data set contains the likelihood (L) value for each accepted set of parameters. Since we are intersted only in constraints of $N_{\rm eff}$, we find the maximum likelihood ($L_{N_{\rm eff}}$) value as a function of $N_{\rm eff}$ over the parameter space sampled in the chain, without regard to the values that the rest of the parameters have. To perform this maximization in practice we bin in $N_{\rm eff}$ with a bin width of 0.5. We verified that the results do not depend strongly on the choice of binning (as long as bins are not so small that they give a very noisy estimate and not so large that they erase any signal). We obtain the profile likelihood ratio by considering $\ln\left(L_{N_{\rm eff}}/L_{\rm max}\right)$ as a function of $N_{\rm eff}$; where $L_{\rm max}$ is the maximum likelihood in the entire chain.

Then, always following [10], we use the pseudo-chisquare defined as $\ln\left(L_{N_{\rm eff}}/L_{\rm max}\right) = 1/2\,\chi^2$, so that $\Delta\ln\left(L_{N_{\rm eff}}/L_{\rm max}\right) = 0.5$ and $\Delta\ln\left(L_{N_{\rm eff}}/L_{\rm max}\right) = 2$ correspond to the 68.3% and 95.4% confidence regions respectively. Finally we report our results in terms of the probability for $\Delta N_{\rm eff} = N_{\rm eff} - 3.04$ for each data set considered. The correspondence between χ^2 intervals and probability is done - as it is customary in this context- assuming Gaussian statistics; the confidence intervals we report therefore could be slightly underestimated because of our Gaussian assumption.

We consider the following data-set combinations: Wilkinson Microwave Anisotropy Probe 7-year data [19, 20] (WMAP) in combination with a Hubble constant prior of [21], WMAP in combination with the baryon acoustic oscillation constraints from the Sloan Digital Sky Survey data release 7 (SDSS DR7) luminous red galaxies (LRG) [22] (WMAP+BAO), WMAP in combination with the SDSS DR7 LRG halo power spectrum of Ref.[23] (WMAP+LRG), and the luminosity distance measurements of type 1A supernovae of Ref. [24] (SN). We also consider high resolution CMB experiments for which data, angular power spectra in the multipole range of interest and likelihood functions were released publicly at the time of writing: ACBAR[25] and ACT [12]. In our analysis we keep the Helium mass fraction (Y_P) fixed to the concordance value. Some analyses [11, 12] allow Y_P to vary. Adding yet another free parameter in the analysis opens up extra degeneracies (in particular $N_{\rm eff}$ and Y_P are anti-correlated), amplifying and possibly exacerbating the prior effects that, we suspect, may "bias" the marginalized posteriors.

In our chains we use a flat prior on H_0 rather than a flat prior on the angular diameter distance to the last scattering surface as in the standard CosmoMC implementation. In fact the fitting formula used by the code to compute the angular diameter distance is accurate for standard number of neutrino species but it is not otherwise. In practice, using a flat prior on this quantity is equivalent to using a non-flat and somewhat complex prior on the true angular diameter distance for non-standard number of neutrinos. This effect goes away when using a flat prior on H_0 and bypassing this analytic fit to the angular diameter distance (see e.g., [14] for discussion on this). For our specific application, using the profile likelihood the choice of prior does not matter and indeed we have verified that by running chains with both a flat prior on H_0 and on the angular diameter distance for some selected cases: the profile likelihood ratio results are indistinguishable, while the marginalized posteriors are not.

III. RESULTS AND CONCLUSIONS

We begin by considering WMAP7 CMB data, both alone and in combination with large-scale structure and supernova datasets. A compilation of the constraints from the different datasets we considered is shown in figure 1. Note that for all datasets, although the maximum likelihood value is always at $\Delta N_{\rm eff} > 0$, $\Delta N_{\rm eff} = 0$ is always well within the 95% error bars. All these combinations have the WMAP7 data in common are are hence not at all statistically independent, so it is not surprising that if in the WMAP data alone the maximum likelihood value is $\Delta N_{\rm eff} > 0$, this persists in all combinations. As an illustrative example on how these error-bars were derived from the profile likelihood ratio, in Fig. 2 we show the probability curve obtained from the profile likelihood ratio for the combination WMAP7+LRG. This can be compared directly with the results of [10].

We then consider the constraints from data-sets combination involving WMAP7 and higher-resolution CMB experiments.
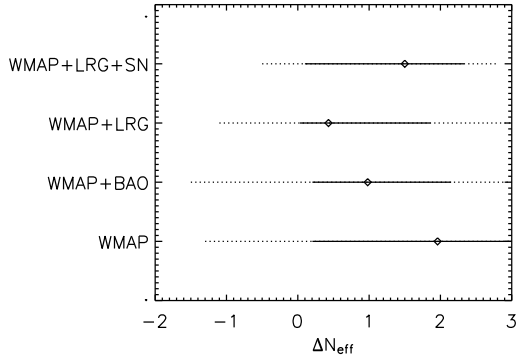
FIG. 1: One and two σ constraints on the number of additional neutrino species from several datasets including combinations of WMAP7, LRG, BAO and SN. Note that the value corresponding to no extra species, $\Delta N_{\rm eff} = 0$, is always well within the 2-σ interval and always very close to the 1-σ interval.
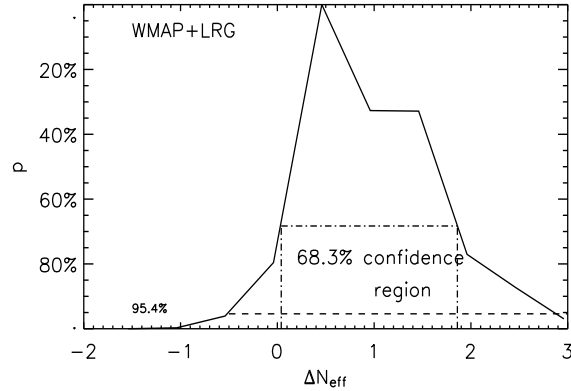

FIG. 2: As an illustrative example we show the probability curve obtained from the profile likelihood ratio for the data combination WMAP+LRG. The curve is jagged because of the binning procedure used to compute the profile likelihood. We have checked that changing the bin size (within reason) only slightly changes the shape of the curve and does not change the inferred confidence regions.
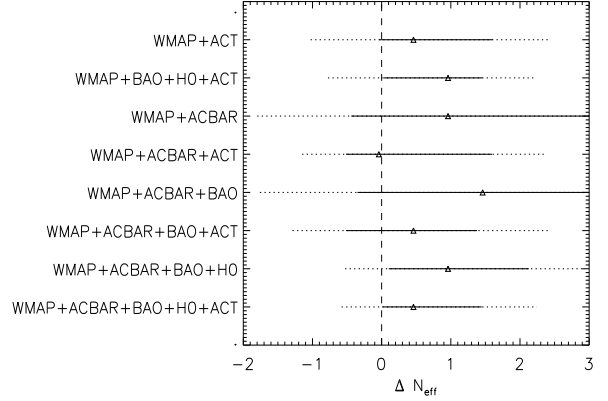

FIG. 3: Overview of one and two σ constraints on extra number of neutrino species from several dataset combinations involving WMAP7 in combination with higher resolution CMB experiments. Note that the point corresponding to no additional relativistic species, $\Delta N_{\rm eff} = 0$, is always well within the 2-σ interval and in few cases within the 1-σ interval. the fact that all constraints seems to be systematically shifted (although by less than 1-σ) towards $\Delta N_{\rm eff} > 0$ is due to the fact that these are *not* independent constraints: the WMAP data, which has a lot of statistical power, is common to all combinations of datasets.
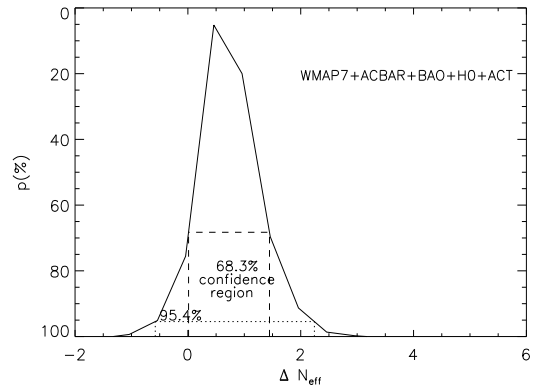

FIG. 4: As an illustrative example we show the probability curve obtained from the profile likelihood ratio for one of the data combinations.

The constraints from the different dataset combinations are summarized in Fig. 3 and an illustrative example of probability curve for one combination is shown in Fig. 4. Again, the standard value $\Delta N_{\rm eff} = 0$ is always well within the 95.4% confidence region and in many cases within the 1-σ region.

If we had looked at the marginalized posterior rather than the profile likelihood, the following data-set combination would have given an almost 2-σ indication of $\Delta N_{\rm eff} > 0$: WMAP7, WMAP7+BAO+H0, WMAP7+LRG+H0 (see [16]). Recall also that looking at the marginalized posterior, for the combinations CMB+H0+SN+BAO and CMB+H0+SN+LSS, [9] find a 2-σ signal for $\Delta N_{\rm eff} > 0$ and that [11] find a 2-σ signal for $\Delta N_{\rm eff} > 0$ for WMAP7+

ACT+ACBAR+BAO+H0, although using different priors and slightly different parameter sets (i.e. the treatment of Y_P).

We can also try to compare with [8] and a more recent work [26]. In these references the authors also allow the neutrino masses to vary. Current cosmological data do not show any evidence for a non-zero neutrino mass and do not have enough statistical power to detect a sum of neutrino masses (Σ) below 0.3 eV (e.g., [10, 27]). Including both $N_{\rm eff}$ and the sum of neutrino masses as free parameters would add new degeneracies and exacerbate further the prior volume effect in the marginalized posterior distributions, especially

since $N_{\rm eff}$ and Σ are positively correlated. To compare with our analysis we need to consider their constraints for $\Sigma = 0$. They find still almost 2-σ indication of $\Delta N_{\rm eff} > 0$ [29]. Our analysis seems to indicate that this evidence may be driven by prior volume effects.

To summarize, the effective number of neutrino species parameterizes any non-standard early-Universe expansion rate. Evidence for $\Delta N_{\rm eff} \neq 0$ could be interpreted as extra (sterile) neutrino species but also as any light particle that does not couple to electrons, the decay of dark matter particles, early quintessence and other phenomena. Recent cosmological analyses carried out in the Bayesian framework have reported hints of $\Delta N_{\rm eff} > 0$; extra neutrino species could explain some recent claims by particle physics experiments (LSND, MiniBoone). In the Bayesian framework, when considering a high-dimensional parameter space as in this case, especially if the data-sets are not highly constraining and cosmological degeneracies are present, the reported marginalized confidence regions on a parameter can depend strongly on the prior choice and on prior volume effects on other parameters. If a result of cosmological analysis, such as evidence for $\Delta N_{\rm eff} > 0$, turns out to be driven by any of these effects, it should be interpreted with extra care. We have presented a way to make the cosmological analysis as prior-independent as possible; we borrowed from the frequentist approach the so-called generalized likelihood ratio to report confidence intervals. We have considered a suite of cosmological data sets and data sets combinations and found that prior-independent confidence

intervals for $\Delta N_{\rm eff}$ do not show any evidence of additional effective neutrino species. Our findings seems to indicate that any evidence for $\Delta N_{\rm eff} > 0$ may be driven by prior effects. As better data become available, the likelihood should overcome the prior and the posterior should become nearly Gaussian, so that this effect should gradually disappear. In the meantime we advocate the use of the generalized likelihood ratio as a useful check of how dependent cosmological results are on the choice of priors.

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[28] In the standard scenario $N_\nu = 3$ and $N_{\rm eff} = 3.04$.
[29] Recall that the 1-σ joint contour in 2 dimensions is much closer to the 2-σ marginalized in 1-dimension than to the 1-σ one.
[30] Lecture notes are available at http://bccp.lbl.gov/beach_program/presentations11.html